CHIZIQLI TENGLAMALAR SISTEMASINI MATRITSA USULIDA YECHISH

Azimov Sherzod Shakarovich, Boboboyev Zafar Sulaymonovich
Muhammad al-Xorazmiy nomidagi Toshkent axborot texnologiyalari universiteti Samarqand filiali akademik litseyi o'qituvchilari
Kenjayev Sharof Shermuxamatovich
Shahrisabz "Temurbeklar maktabi" harbiy-akademik litseyi o'qituvchisi

n ta no'malumli n ta chiziqli tenglamalar sistemasi berilgan bo'lsin:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = y_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = y_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = y_n \end{cases} \qquad (1)$$

(1) sistemaning noma'lumlari oldidagi koeffitsiyentlardan matritsa hamda ozod hadlar va noma'lumlardan ustun matritsalar tuzib, ularni quyidagicha belgilaymiz:

$$A = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{pmatrix}, \ B = \begin{pmatrix} y_1 \\ y_2 \\ \ldots \\ y_n \end{pmatrix} \text{ va } X = \begin{pmatrix} x_1 \\ x_2 \\ \ldots \\ x_n \end{pmatrix} \qquad (2)$$

Matritsalarni ko'paytirish qoidasidan foydalanib, (1) sistemani quyidagicha yozish mumkin:

$$A \cdot X = B \qquad (3)$$

Agar $\boldsymbol{detA = 0}$ (ya'ni $\boldsymbol{A}$ matritsa xosmas) bo'lsa, $\boldsymbol{A^{-1}}$ mavjud. U holda, (3) ning ikkala tomonini $\boldsymbol{A^{-1}}$ ga ko'paytiramiz:

$$A^{-1} \cdot (A \cdot X) = A^{-1} \cdot B \qquad (4)$$

(4) tenglikni quyidagicha yozish ham mumkin:

$$(A^{-1} \cdot A) \cdot X = A^{-1} \cdot B \qquad (5)$$

Matritsa bilan unga teskari matritsaning ko'paytmasi birlik matritsaga, A matritsa bilan A^{-1} teskari matritsaning ko'paytmasi E birlik matritsaga, E birlik matritsaning X ustun matritsaga ko‘paytmasi natijasida, X ustun matritsa hosil bo‘lishini, ya’ni:

$A^{-1} \cdot A = E$ va $E \cdot X = X$

ekanligini hisobga olsak, quyidagi tenglik kelib chiqadi:

$$X = A^{-1} \cdot B \qquad (6)$$

(6) tenglik $detA \neq 0$ bo‘lganda n noma’lumli n ta chiziqli tenglamalar sistemasi yechimining matritsali ifodasidan iborat.

Misol. Tenglamalar sistemasini matritsa usulida yeching:

$$\begin{cases} x_1 + x_2 + 2x_3 = 4 \\ 2x_1 + x_2 + x_3 = 3 \\ x_1 + 2x_2 + x_3 = 1 \end{cases}$$

Yechilishi: Quyidagi belgilashlarni kiritamiz:

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{pmatrix}; \qquad B = \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix}; \qquad X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

A matritsaning Δ determinantini hisoblaymiz:

$$\Delta = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix} = 1 + 8 + 1 - 2 - 2 - 2 = 4, \quad \Delta = 4 \neq 0.$$

Demak, A matritsaga taskari A^{-1} matritsa mavjud, chunki $detA \neq 0$. A^{-1} ni topish uchun determinantning algebraik to‘ldiruvchisini, so‘ngra esa biriktirilgan $\bar{A}$ matritsani topamiz:

$$A_{11} = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -1; \qquad A_{21} = -\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = 3; \qquad A_{31} = \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = -1;$$

$$A_{21} = -\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = -1; \qquad A_{22} = \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = -1; \qquad A_{32} = -\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = 3;$$

$$A_{13} = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 3; \qquad A_{23} = -\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = -1; \qquad A_{33} = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -1.$$

$$\bar{A} = \begin{pmatrix} -1 & 3 & -1 \\ -1 & -1 & 3 \\ 3 & -1 & -1 \end{pmatrix}; \quad A^{-1} = \begin{pmatrix} -\frac{1}{4} & \frac{3}{4} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & \frac{3}{4} \\ \frac{3}{4} & -\frac{1}{4} & -\frac{1}{4} \end{pmatrix}.$$

(6) formulaga asosan quyidagiga ega bo‘lamiz:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -\frac{1}{4} & \frac{3}{4} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & \frac{3}{4} \\ \frac{3}{4} & -\frac{1}{4} & -\frac{1}{4} \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{4}\cdot 4 + \frac{3}{4}\cdot 3 - \frac{1}{4}\cdot 1 \\ -\frac{1}{4}\cdot 4 - \frac{1}{4}\cdot 3 + \frac{3}{4}\cdot 1 \\ \frac{3}{4}\cdot 4 - \frac{1}{4}\cdot 3 - \frac{1}{4}\cdot 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

Demak, berilgan tenglamalar sistemasining yechimlari quyidagicha ekan: $x_1 = 1$; $x_2 = -1$; $x_3 = 2$

Foydalanilgan adabiyotlar

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