Mellin akslantirishi yordamida matematik fizika masalalarini yechish

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$f(t)$ funksiya musbat t uchun aniqlangan va

$$\int_{0}^{+\infty} |f(t)| t^{\sigma-1} dt < +\infty$$

shartni qanoatlandirsin. $f(t)$ funksiyaning Mellin akslantirishi

$$F(s) = \int_{0}^{+\infty} f(t) t^{s-1} dt, \qquad (s = \sigma + i\tau), \qquad (1)$$

funksiya hisoblanadi [1].

Mellin akslantirishining teskari formulasi quyidagicha

$$f(t) = \frac{1}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} F(s) t^{-s} ds, \qquad (t > 0), \qquad (2)$$

bunda integral S tegislikning mavhum koordinatasiga parallel $\operatorname{Re} s = \sigma$ chiziq bo'ylab olinadi [2].

Mellin akslantirishi yordamida quyidagi chegaraviy masalani yechishni ko'rib chiqamiz [3] :

$$x^2 U_{xx} + xU_x + U_{yy} = 0, \qquad (0 \le x < \infty, 0 < y < 1), \qquad (3)$$

$$U(x,0) = 0, \quad U(x,1) = \begin{cases} A, 0 \le x < 1 \\ 0, x > 1. \end{cases} \qquad (4)$$

bunda $A-$o'zgarmas son.

Mellin akslantirishini $x-$o'zgaruvchi bo'yicha foydalanamiz. Shunda $x-$o'zgaruvchi bo'yicha Mellin akslantirishi quyidagicha bo'ladi:

$$x^2U_{xx} \Rightarrow \int_0^\infty x^2U_{xx}x^{s-1}dx = \int_0^\infty U_{xx}x^{s+1}dx,$$

ikki marta bo'laklab integrallaymiz:

$$\int_0^\infty U_{xx}x^{s+1}dx = \begin{vmatrix} u = x^{s+1} \\ du = (s+1)x^s dx \\ dv = U_{xx}dx \\ v = U_x \end{vmatrix} = \underbrace{U_x x^{s+1}\Big|_0^\infty}_{0} - (s+1)\int_0^\infty U_x x^s dx = -(s+1)\int_0^\infty U_x x^s dx =$$

$$= \begin{vmatrix} u = x^s \\ du = sx^{s-1} \\ dv = U_x dx \\ v = U \end{vmatrix} = -(s+1)\left[\underbrace{x^sU\Big|_0^\infty}_{0} - s\int_0^\infty Ux^{s-1}dx\right] = s(s+1)\int_0^\infty Ux^{s-1}dx = s(s+1)\bar{U}(s,y).$$

$$xU_x \Rightarrow \int_0^\infty xU_x x^{s-1}dx = \int_0^\infty U_x x^s dx,$$

buni bir marta bo'laklab integrallaymiz:

$$\int_0^\infty U_x x^s dx = \begin{vmatrix} u = x^s \\ du = sx^{s-1}dx \\ dv = U_x dx \\ v = U \end{vmatrix} = \underbrace{x^sU\Big|_0^\infty}_{0} - s\int_0^\infty Ux^{s-1}dx = -s\int_0^\infty Ux^{s-1}dx = -s\bar{U}(s,y).$$

Unda

$$U_{yy} = \int_0^\infty U_{yy}x^{s-1}dx = \bar{U}_{yy},$$

bo'ladi. Endi berilgan (4)-chegaraviy shartlarning Mellin akslantirishini topamiz:

$$U(x,0) \Rightarrow \int_0^\infty \underbrace{U(x,0)}_{0}x^{s-1}dx = 0,$$

bundan ko'rinib tu'ribdi $\bar{U}(s,0)=0$ ga teng.

$$U(x,1)\Rightarrow\int_0^\infty U(x,1)x^{s-1}dx=\int_0^1 U(x,1)x^{s-1}dx+\int_1^\infty \underbrace{u(x,1)}_{0}x^{s-1}dx=$$

$$=\int_0^1 \underbrace{U(x,1)}_{A}x^{s-1}dx=A\int_0^1 x^{s-1}dx=A\cdot\frac{x^s}{s}\Big|_0^1=\frac{A}{s},$$

bundan ko'rinib tu'ribdi $\bar{U}(s,1)=\dfrac{A}{s}$ ga teng

Demak, (3)-(4) chegaraviy masala quyidagi ko'rinishga keladi:

$$\begin{cases}\bar{U}_{yy}+s^2\bar{U}=0,\\ \bar{U}(s,0)=0,\bar{U}(s,1)=\dfrac{A}{s}.\end{cases}$$

Bu tenglamaning umumiy yechimi

$$\bar{U}(s,y)=C_1\cos sy+C_2\sin sy, \qquad (5)$$

bo'ladi. Endi berilgan chegaraviy shartlardan C_1,C_2 larni aniqlaymiz,

$$\begin{cases}C_1\cdot 1+C_2\cdot 0=0,\\ C_1\cos s+C_2\sin s=\dfrac{A}{s},\end{cases}$$

bundan,

$$\begin{cases}C_1=0,\\ C_2=\dfrac{A}{s\sin s}.\end{cases}$$

Topilgan C_1,C_2 larni (5) ga olib borib qoyamiz va quyidagi umumiy yechimga ega bo'lamiz:

$$\bar{U}(s,y)=\frac{A\sin sy}{s\sin s}.$$

Endi teskari Mellin akslantirishidan foydalanib $U(x,y)$ ni topamiz.

$$U(x,y)=\frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty}\frac{A\sin sy}{s\sin s}x^{-s}ds.$$

Foydalanilgan adabiyotlar

[1]. Золотарев В.М. Преобразование Меллина-Стильтеса в теории вероятностей. «Теория вероятностей и ее применение». Т. 2, 1957, № 4, с. 444-469.

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[3]. Салохитдинов М.С., Исломов Б.И. Математик физика тенгламалари фанидан масалалар тўплами.- Т.:Mumtoz soz, 2010.